Laser Burnthrough Time Reduction Due to Tangential Airflow—An Interpolation Formula

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This paper contains an investigation of the influence of a tangential gas flow on the time required for a laser beam of given intensity to burn through a metal sheet. From a consideration of the hydrodynamics of the liquid metal-gas interface and the time required for an entrained liquid droplet to vaporize, an interpolation formula is derived relating the time required for burnthrough at a given freestream Mach number to that required in the absence of tangential flow, or for flow rates less than the critical velocity at which melt droplets are entrained. The interpolation formula so derived depends only on the thermodynamic properties of the melt, the hydrodynamic properties of the boundary layer and the size of the spot irradiated by the laser beam. The formula predicts an intermediate velocity for which the time for burnthrough is a minimum. Physical explanation of the minimum is given in the text. For limiting cases of very large laser spot size, the reduction in burnthrough time disappears.

Introduction

WHEN a high intensity laser beam is incident upon a metal sheet, the surface temperature is raised to the melting point, typically in a very short time. The thin layer of melt so formed continues to increase in temperature and begins to vaporize. The melt layer deepens until it spans the thickness of the specimen and the melt continues to vaporize. When the melt front reaches the opposite side of the material, we say the material has "burned through." The time required to do this is roughly equal to the product of the mass vaporized and the heat of the over-all transition per unit mass divided by the rate of energy deposition from the laser.

The presence of an airstream flowing tangential to the irradiated surface, if the freestream velocity is sufficiently high, results in the entrainment of some of the melt. The entrained droplets are accelerated by the airstream and are subsequently in part removed from the laser beam and in part vaporized before removal. When a considerable fraction of the entrained liquid escapes the beam without vaporization, the energy requirements and, correspondingly, the laser burnthrough time are reduced.

Because the hydrodynamically unstable surface of the melt is neither plane nor stationary, the difficulties of obtaining a satisfactory numerical solution can be readily appreciated. However, it is possible, by relatively simple methods, to construct an interpolation formula relating the burnthrough time required without airflow to that when a destabilizing airflow is present. The technique is based on comparing the time required for entrainment and removal (from the laser beam) of a liquid droplet to the time required for the beam to vaporize the droplet and employs essentially the same method as has been used with success in the theory of film boiling.[1,2] The result is an expression for the burnthrough time required at any freestream Mach number in terms of the static burnthrough time as well as parameters describing the melt-air interface. Ostensibly the result is restricted to circumstances where the spot size (radius of the irradiated area) is large compared to the thickness of the material and the intensity profile of the laser beam is nearly uniform, but the error induced by relaxing these constraints is probably small. No attempt has been made to include combustion of the melt as occurs when heating some metals in air; however, the droplet sizes and kinematics predicted below are important aspects of the combustion process.

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Melt Entrainment

The basic results concerning the stability of the interface between two fluids are well known.[3] Here we briefly note the necessary results. Figure 1 shows the situation schematically. Air is flowing over the irradiated (and melting) surface with freestream velocity U. The thin layer of molten liquid is characterized by density ρ and surface tension σ. The mean, or quasi-steady thickness of the melt layer is h. The atmospheric density is ρ'. For an assumed interface disturbance of the form

$$\eta(x,t) = a\,e^{i(kx-\omega t)} \tag{1}$$

one obtains the (Kelvin-Helmholtz) allowed values for ω

$$\omega = \frac{\rho' U k}{\rho ctnh(kh)+\rho'} \pm \frac{1}{\rho ctnh(kh)+\rho'}\{[\rho ctnh(kh)+\rho'] \times [(\rho-\rho')gk+\sigma k^3] - \rho\rho' U^2 k^2 ctnh(kh)\}^{1/2} \equiv u \pm iv \tag{2}$$

Here k is the wave number and ω the frequency. For real values of v (negative values of the radicand), η grows or decays as

$$\eta = a\,e^{\pm vt} \tag{3}$$

where for $\rho' \ll \rho$

$$v = [\tanh(kh)]^{1/2}\left[\frac{\rho' U^2 k^2}{\rho} - \left(gk + \frac{\sigma k^3}{\rho}\right)\right]^{1/2} \tag{4}$$

Figure 2 shows a sketch of $F(k) = v^2/\tanh(kh)$. The values of v corresponding to growing waves are given by $v = +[\tanh(kh)]^{1/2}[F(k)]^{1/2}$ in the first quadrant. Examination of the curves shows that the interface is characterized by a range of unstable wave numbers $k_- < k < k_+$ where

$$k_\pm = \frac{\rho' U^2}{2\sigma}\left[1 \pm \left(1 - \frac{4\rho g\sigma}{(\rho' U^2)^2}\right)^{1/2}\right]$$

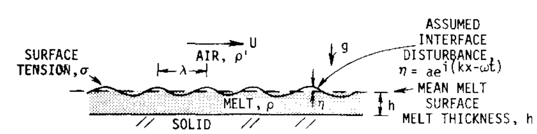

Fig. 1 Illustrating aerodynamic instability of a melt layer formed by laser radiation.

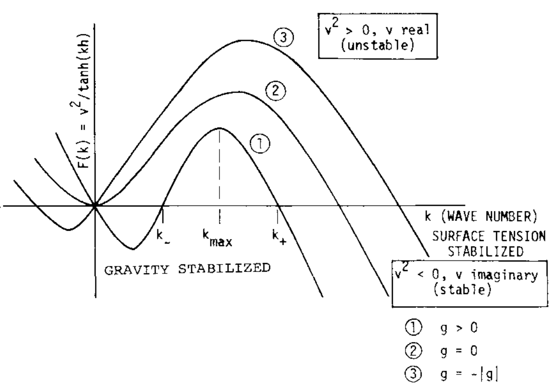

Fig. 2 Regions of stability and instability of the melt surface.

and a maximum growing wavelength, k_{max}. Also for $g > 0$, there is a critical freestream velocity below which the surface is absolutely stable. This velocity is given by

$$U_c = [3\rho g\sigma/\rho'^2]^{1/4} \tag{5}$$

When $g = 0$ or $-|g|$, $U_c = 0$. Table 1 gives computed values of U_c for several metals.

The values of $k_\pm$ are independent of the melt thickness h while that for k_{max} is only weakly dependent on it. This can be shown by calculating k_{max} for two limiting cases, $h \to 0$ and $h \to \infty$. In the former case

$$k_{max} = \frac{3\rho' U^2}{8\sigma}\left[1 + \left(1 - \frac{32\rho g\sigma}{9(\rho' U^2)^2}\right)^{1/2}\right] \tag{6a}$$

While in the latter

$$k_{max} = \frac{\rho' U^2}{3\sigma}\left[1 + \left(1 - \frac{3\rho g\sigma}{(\rho' U^2)^2}\right)^{1/2}\right] \tag{6b}$$

the difference being only about 12%. We will use the latter value explicitly assuming that the thin film of melt can, from a hydrodynamic viewpoint, be treated as infinitely thick. As we have seen previously, this makes little difference as far as k_{max} is concerned, but the growth rate $v(k)$ depends on h as $[\tanh(kh)]^{1/2}$.

Table 1 Critical velocity at sea level

Metal	Surface tension (dyne/cm)	Density (g/cm^3)	Critical velocity (cm/sec)
Al	840	2.71	1.45×10^3
Ti	1600	4.5	1.95×10^3
Fe	1700	7.9	2.28×10^3

From the values of U_c given in Table 1 we see that $k_{max}(U_c) \simeq 1$ so $[\tanh(kh)]^{1/2} \simeq h^{1/2}$ since h is in fact small. However, k_{max} increases as U^2 so that for somewhat higher velocities $\tanh(kh) \simeq 1$ even for quite small values of h. With this assumption we can use the Kelvin-Helmholtz results for $h \to \infty$ for computing k_{max} and $v(k_{max})$.

As pointed out earlier, our goal is to compare the time for entrainment and escape of a droplet from the laser beam with the time required for the beam to vaporize the droplet. To do so we must characterize the entrainment time, the escape time and the mean size of the droplet at entrainment. We assume that the mean entrainment event corresponds to k_{max}. This is altogether reasonable since disturbances at this wave number grow most rapidly. Further, the assumption has been found to hold quite well for vapor bubbles formed in film boiling.[4] If this is the case, we may take the mean diameter of the droplets as $D = \lambda_{max}/2 = \pi/k_{max}$. We will also assume that a characteristic time for entrainment

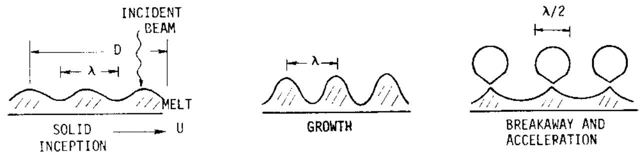

Fig. 3 Illustrating the growth and entrainment of liquid metal droplets.

is $\tau_e = 1/v(k_{max})$. Figure 3 may aid in understanding the assumptions. The unstable waves grow on the melt surface ultimately breaking away into droplets. Their size must in any case be close to one-half the wavelength. The e folding time for the growth of the waves during their early linear phase is $1/v$. The total time for growth and breakaway is larger than this by a factor the order of one. After breakaway the droplets experience rapid acceleration by the freestream and leave the beam in a removal time on the order of $\tau_t \simeq d/U$ where d is a dimension on the order of the laser beam diameter. See Appendix A for a discussion of the droplet acceleration time.

Substituting k_{max} [Eq. (6b)] into Eq. (4) we have

$$\tau_e \simeq \frac{1}{v(k_{max})} = \left(\frac{27\rho\sigma^2}{4\rho'^3}\right)^{1/2}\frac{1}{U^3} \tag{7}$$

$$D = \left(\frac{3\pi\sigma}{2\rho'}\right)\frac{1}{U^2} = \frac{\lambda_{max}}{2} \tag{7a}$$

for the special case where $g = 0$. [These equations are also a good approximation when $g \neq 0$ improving for $U > U_c$; see Eq. (6b) and note that the quantity $3\rho g\sigma/(\rho' U^2)^2 = 1$ when $U = U_c$, and decreases as the inverse fourth power of U for $U > U_c$.] We note that the entrainment time grows very small for large freestream velocity U, indicating that in this case, the controlling time will be that required for the droplets to accelerate and escape the beam. Equation (7a) indicates that the droplet diameter decreases rapidly with freestream velocity so that at appreciable velocities ($U \gg U_c$) the entrainment is in the form of a fine mist.

Interpolation Formula for Burnthrough Time

When the freestream velocity is less than critical ($U < U_c$) the interface is stable and no entrainment occurs. In this case the burnthrough time, t_{bt}, is the sum of the times required for heating and melting the solid, t_l, and the time required for heating and vaporizing the melt, t_{lv}. This is strictly true only when a uniform hole is burned through the material. Thus

$$t_{bt} = t_l + t_{lv} \tag{8}$$

The times t_l and t_{lv} are computed from the measured burnthrough time and thermodynamic data. Thus $\Delta h_m/\Delta h_t = t_l/t_{bt}$; $\Delta h_v/\Delta h_t = t_{lv}/t_{bt}$ where Δh_m is the enthalpy difference between the initial (preirradiation) state of the material and the melted state, Δh_v, that between the melted state and vaporization and Δh_t is the sum of the two.

When entrainment is present ($U > U_c$) some melt droplets will escape the beam before vaporization is complete. In this case we write

$$t_{bt} = t_l + f t_{lv} \tag{9}$$

where f is the fraction of the melt heated and vaporized before leaving the irradiated area. We now assume that f is proportional to the ratio of two times; the mean residence time of an entrained droplet in the beam and the mean time required by the beam to heat and vaporize that droplet. Enhancement, or reduction in required burnthrough time, occurs when $f < 1$, in other words, when the droplets escape without being completely vaporized. Whenever $f \geqq 1$ we must put $f = 1$ because $f > 1$ describes all cases where the residence time is greater than that required for vaporization and there is no enhancement.

For a droplet of volume dV the time required for heating and vaporization under constant irradiation of intensity I is approximately $\tau_v = dV\rho(\Delta h_v)/\alpha_\lambda IA$. Here ρ is the melt density, α_λ is the absorptivity at wavelength λ, (the laser wavelength) and A is the droplet area exposed to the beam. Then

$$f \propto \frac{\tau_e + \tau_t}{(dV\rho/\alpha_\lambda IA)\Delta h_v} \tag{10}$$

For spherical droplets $dV/A = \frac{2}{3}R = \lambda_{max}/6$ and thus depends on the mean size of the entrained droplets. Including the constants in the constant of proportionality, we may write ($\lambda_{max} = 2\pi/k_{max}$)

$$f = C(\tau_e + \tau_t)k_{max} \tag{11}$$

The constant C is determined from the condition $f = 1$ when $U = U_c$. This condition along with the previously derived expressions for τ_e, τ_t, and k_{max} gives

$$f = \left(\frac{U_c}{U}\right)\left[\frac{1+\frac{2}{3}(\rho'/3\rho\sigma^2)^{1/2}\,d\rho' U^2}{1+\frac{2}{3}(\rho'/3\rho\sigma^2)^{1/2}\,d\rho' U_c^2}\right] \tag{12}$$

where we have again assumed gravitational effects are negligible. Then the interpolation formula relating burnthrough time at velocity U (or Mach number M) to the static burnthrough time is

$$t_{bt} = t_l + \left(\frac{U_c}{U}\right)\left[\frac{1+\frac{2}{3}(\rho'/3\rho\sigma^2)^{1/2}\,d\rho' U^2}{1+\frac{2}{3}(\rho'/3\rho\sigma^2)^{1/2}\,d\rho' U_c^2}\right]t_{lv} \tag{13}$$

$$= t_l + \left(\frac{M_c}{M}\right)\left[\frac{1+\frac{2}{3}(\rho'/3\rho\sigma^2)^{1/2}\,d\rho' c^2M^2}{1+\frac{2}{3}(\rho'/3\rho\sigma^2)^{1/2}\,d\rho' c^2M_c^2}\right]t_{lv} \tag{13a}$$

Equations (12) and (13) have several interesting properties. The most interesting one is they predict a freestream velocity U for which f is minimum. This value of U is found from Eq. (12) as

$$U_{min} = \left[\frac{3}{2}\right]^{1/2}\left(\frac{3\rho\sigma^2}{\rho'^3}\right)^{1/4}\frac{1}{d^{1/2}} \tag{14}$$

Representative values for $d = 1$ cm (1 cm laser beam radius) are $\bar{U}_{min} = 9000$ cm/sec ($M = 0.3$) for aluminum and 15,400 cm/sec ($M = 0.47$) for titanium. The value of ρ' is taken as the sea level value, 1.22×10^{-3} g/cm^3. Equation (14) shows that U_{min} is inversely proportional to the spot size to the one-half power. For large values of d, U_{min} decreases and approaches U_c.

The existence of a velocity at which the required burnthrough time is a minimum can be explained as follows. For freestream velocities just over the critical ($U > U_c$) the entrainment time τ_e is larger than the transit time τ_t. Quantity τ_e decreases as $1/U^3$ so that the total removal time $\tau_e + \tau_t$ decreases rapidly at first. The rapid reduction in removal time causes a correspondingly rapid decrease in f and hence t_{bt}. From then on the removal time decreases as $1/U$ whereas the droplet vaporization time decreases as $1/U^2$. Ultimately the mean removal time equals the vaporization time at which point enhancement disappears.

Calculated Burnthrough Time Reduction—Discussion

This behavior is exhibited in the calculated curves of Figs. 4 and 5 showing the burnthrough time of selected samples of aluminum and titanium, respectively. The total burnthrough time was selected arbitrarily as 1.0 sec for aluminum and 3.6 sec for titanium. The curves show the expected decrease in burnthrough time as a function of Mach number. Freestream conditions correspond to sea level. The spot size (d) used was 1 cm. The quantities t_l and t_{lv} were calculated using data from Ref. 5. To make the calculations, some assumptions concerning the thermodynamic state of the droplet at entrainment is necessary since we do not know the surface temperature. Of the total energy change, however, only a small fraction is contained in the sensible heat of the liquid melt. Most is invested in the heat of fusion and vaporization. Therefore, it makes little difference whether we assume the melt surface is at the liquid melting temperature or at the other extreme the boiling temperature. Figures 4 and 5 are based on the former.

In the case of titanium the sensible heat of the solid plus the heat of fusion totals 1.72×10^4 cal/g at m. The sensible heat of the melt between melting and boiling is 1.16×10^4 cal/g atm, while the heat of fusion is 1.01×10^5 cal/g atm. These data are

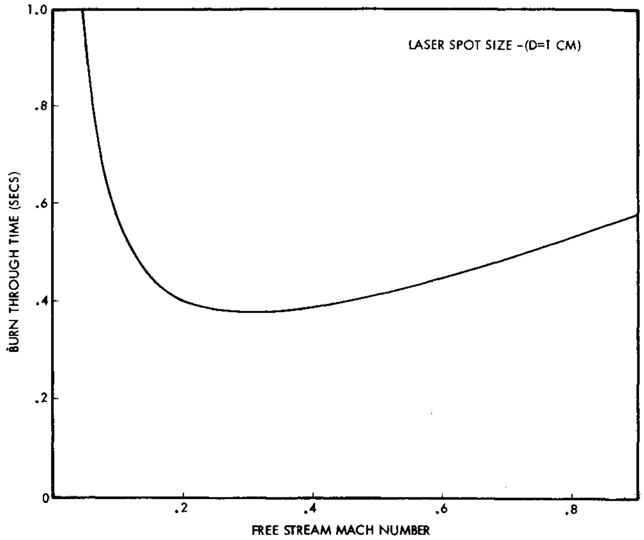

Fig. 4 Computed enhancement for aluminum.

from Ref. 6. Consequently, uncertainty in the surface temperature at entrainment can cause an error of no more than about 12%. Note also that for titanium sensible heat plus heat of fusion of the solid constitutes about 13% of the total heat of transition from solid to vapor. According to Fig. 5, for a 1 cm spot size, the minimum time required is about 33% of the static time, indicating that for this spot size a large fraction of the melt can be removed without vaporization. The fraction is the larger, the smaller is the spot size; however, the present analysis is not accurate for too small spot sizes because of the assumption of spot size much greater than the specimen thickness.

Figure 6 shows the effect of spot size on burnthrough time reduction for aluminum. Again the total static burnthrough time was taken as 1 sec. Note the expected decrease in enhancement as spot size is increased because of the increased residence time of the droplets in the irradiating beam.

Finally, we would like to make a remark concerning the applicability of the formulas [Eqs. (12) and (13)] to situations where the gravity vector is not along the incident laser beam. This assumption ($g > 0$) is implicit in the derivation (even though we have set $g = 0$ in the final result) through the use of a nonzero value for the critical velocity U_c. When the laser beam is perpendicular to the gravity vector (irradiation of a plate standing on edge) or in direction opposite to gravity (irradiation of the

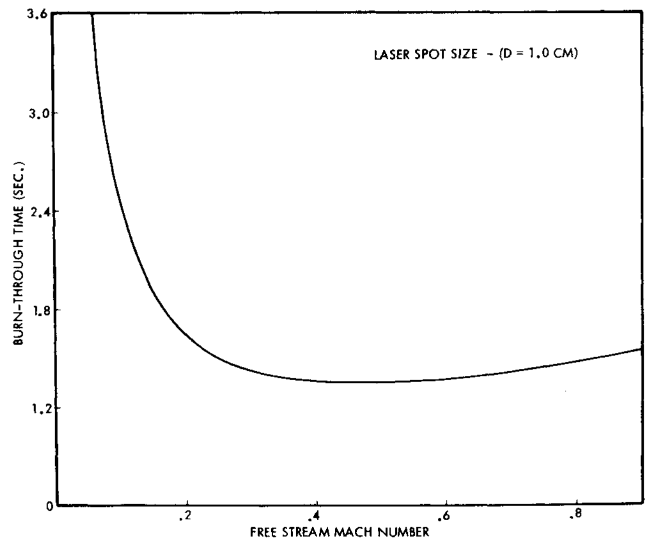

Fig. 5 Computed enhancement for titanium.

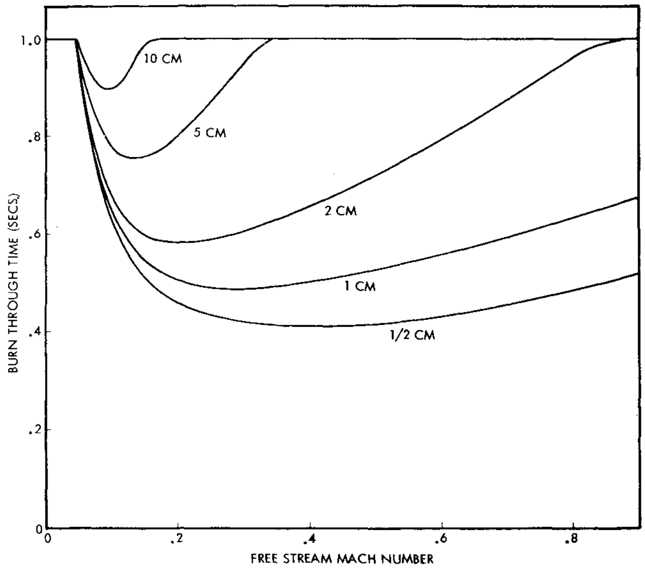

Fig. 6 Effect of beam spot size on aerodynamic enhancement for aluminum.

bottom surface), $U_c = 0$ and the formulas fail. However, a little consideration of the physics shows that there is a finite critical velocity for these cases also although not given by Eq. (5). For example, when $g = 0$ and U is very small, we find that $k_\pm$ and $k_{\max}$ are also very small and $\lambda_\pm$, $\lambda_{\max}$ very large, much larger than any given laser spot size. Thus the entrainment process (droplets much smaller than the laser spot size) upon which the analytical model is based does not really occur until some higher freestream velocity is reached, one which corresponds to a growing wavelength on the order of the laser spot size. With this reservation Eqs. (12) and (13) are then valid. A different value of U_c may be required for each spot size d. When $g = -|g|$ we have

$$k_{\max} = [\rho\,|g|/\sigma]^{1/2}; \quad \lambda_{\max} = 2\pi[\sigma/\rho\,|g|]^{1/2}$$

at $U = 0$ and the situation is more complex since $\lambda_{\max}$ is, in this case, on the order of a few centimeters for liquid metals of interest ($\sigma \simeq 10^3$ dynes/cm). For small spot sizes $[d \ll k_{\max}(U = 0)]$ there is again an effective critical velocity and Eqs. (12) and (13) apply.

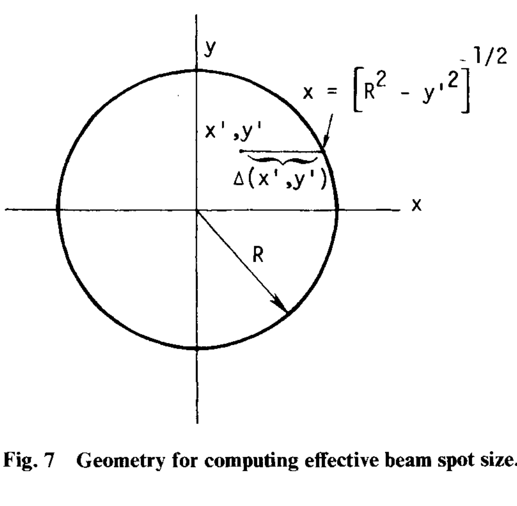

Fig. 7 Geometry for computing effective beam spot size.

Appendix A—Droplet Acceleration Time

Equation (12) is based on the simple approximation d/U for the mean transit time of a droplet from its point of entrainment to the beam perimeter. This neglects acceleration and the shape of the beam-material interaction region. It is strictly correct only for a rectangular spot shape where d is then one half the dimension of the rectangular in the direction of the freestream velocity. The usual case is a roughly circular spot or interaction region. In the following, a brief discussion of the error by the use of the simplified equation will be attempted.

The velocity of a droplet of constant size under acceleration by the freestream is given by

$$v(t) = \frac{\frac{3}{8}(C_D/R)U^2 t}{1 + \frac{3}{8}(C_D/R)Ut} \tag{A1}$$

Here C_D is the drag coefficient (assumed constant), R is the droplet radius, and t is the time after entrainment. When t is much larger than $8R/3C_D U$, $v(t) = U$. Let us compare this time to τ_e, the entrainment time. Using $R = \lambda_{\max}/4$ and $C_D \simeq 1$, $\tau_a \simeq \lambda_{\max}/U$ and

$$\tau_a/\tau_e \simeq (\rho'/\rho)^{1/2} \tag{A2}$$

For metals in air the ratio is of the order 10^{-3}. We conclude that the acceleration time is about an order of magnitude less than the entrainment time. We have shown that for freestream velocities much greater than the critical, τ_e becomes very small and much less than the mean transit time as calculated from d/U. Therefore acceleration times can be neglected except for velocities just greater than the critical velocity and no large error is introduced by computing the transit time as some characteristic dimension divided by the freestream velocity.

Appendix B—The Equivalent Spot Size for a Uniform Intensity Laser Beam

Based on this result, we can compute the mean transit time for a circular spot assuming a uniform laser intensity profile. Referring to Fig. 7 we consider a droplet entrained at the point (x', y'). It travels the distance $\Delta = x - x' = [R^2 - y'^2]^{1/2} - x'$ in time $t = \Delta/U$. The mean value of t is obtained by integrating over the circle and dividing by the area A. The mean of t is τ_t.

$$\bar{t} = \tau_t = (1/AU)\int_A \Delta(x', y')\,dx'\,dy' \tag{B1}$$

The integration is straightforward and gives

$$\tau_t = 8R/3\pi U = 0.845(R/U) \tag{B2}$$

Thus for a circular spot and a uniform beam profile, the quantity d used previously is equal to $0.845R$ where R is the radius of the spot.

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